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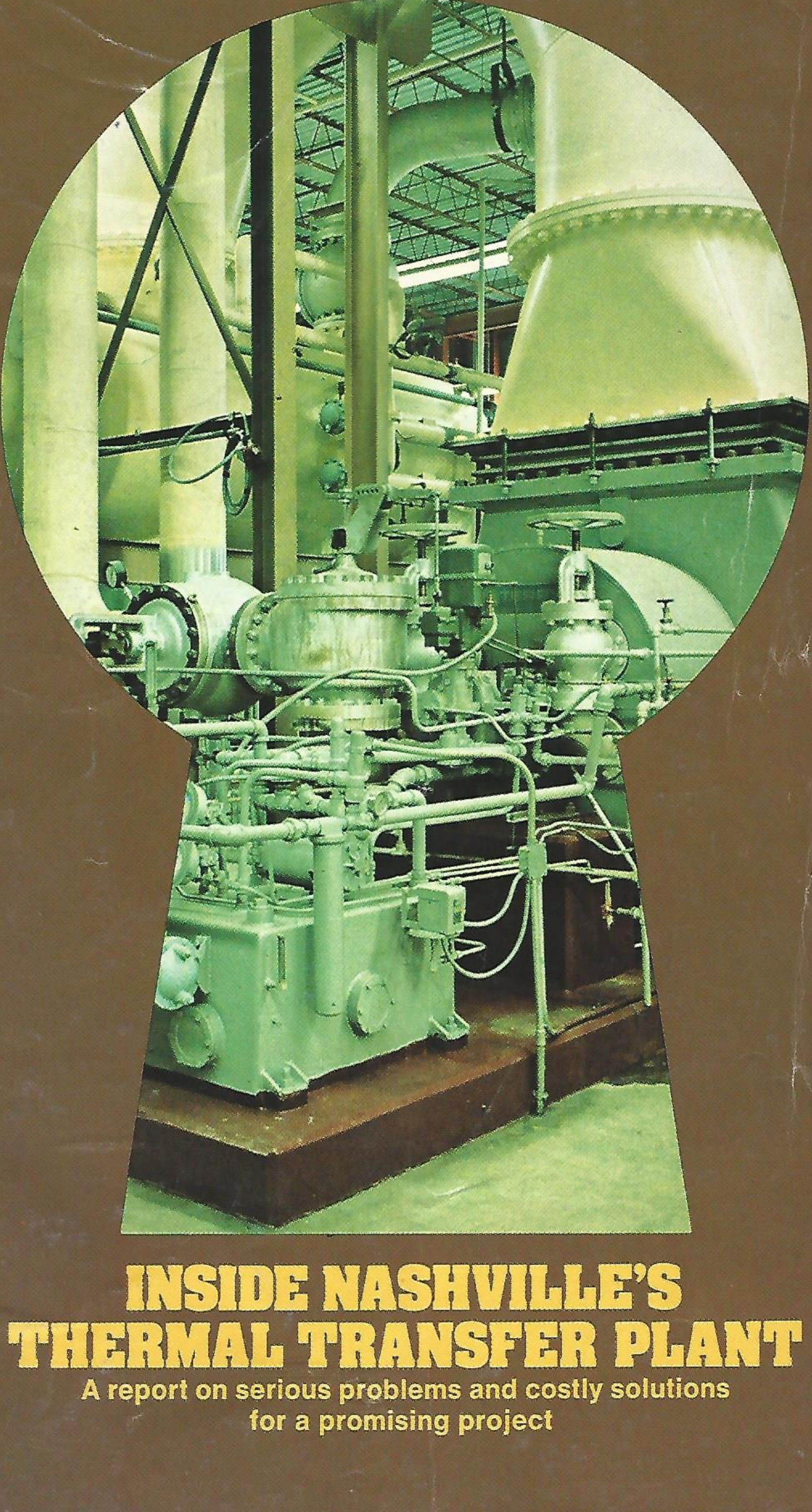

INSIDE NASHVILLE'S THERMAL TRANSFER PLANT

A report on serious problems and costly solutions for a promising project

Finding The Best Angle And Spacing For Solar Collectors

A computer program formulates optimum collector angles and spacing for maximum solar gain in typical installations

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☐ *Several recent projects we have worked on used flat-plate solar collector systems, most often placed on the roofs but sometimes on the side walls of the buildings. Subsequently, we made a careful study to determine the optimum collector angles for various installations. As a result, the following short computer program was developed.*

The ideal situation for receiving solar energy with conventional flat-plate collectors is to rotate the panel so that it follows the sun. This is impractical, however, for such applications as large office or hi-rise buildings where the collectors need to be safely fastened and secured against wind load and the installation cost is an important factor. The energy that fixed collectors receive, therefore, depends very much on their orientation and spacing. Studies* have been conducted to determine the optimum orientation for south-facing panels in the northern hemisphere. Best results are not always obtained by having the panels facing south as in the case where panels are mounted on the east or west faces of the building. Since in these instances the building shades the panels to some extent, the optimum orientation is not south.

Another factor having an influence on the amount of radiation received is the spacing between panels where rows of panels are used. A roof with a large number of panel rows has the capability of receiving a large amount of energy, but there also is considerable shading from the adjacent panels. On the other hand, a small number of rows results in very little shading, but the energy received is decreased because of the comparatively low panel area.

A procedure is outlined below in which optimum panel angles and spacing is determined for typical applications.

Finding optimum angles

The total shortwave radiation reaching the surface of the earth in a year is given by Equation 1:

$$I_t = I_{dn} \times \text{Cos } \theta + I_d + I_r, \text{ Btuh/ft}^2 \qquad (1)$$

Where:

I_{dn} = the direct normal irradiation, Btuh/ft²

θ = the angle between the sun's rays and the panel's surface.

I_d = the diffuse radiation, Btuh/ft²

I_r = the reflected radiation, Btuh/ft²

The direct normal and diffuse radiation are partly determined by a procedure found in ASHRAE's 1972 *Handbook of Fundamentals* (pages 386-400). Reflected radiation, however, is neglected in this study. The direct normal irradiation at the earth's surface on a clear day is:

$$I_{dn} = A/\exp^{(B\text{sine}\,\beta)} \text{ Btuh/ft}^2 \qquad (2)$$

Where:

A = the apparent solar irradiation outside the earth's atmosphere Btuh/ft²

B = atmospheric extinction coefficient

β = the solar altitude

exp = exponential constant 2.718

Values for A and B are taken from research conducted at the University of Minnesota† . The values are seen to be a function of the solar declination. For this study, functional relationships were derived by the use of data-fitting computer subroutines. The functions relate the values of A and B to the particular day of the year.

The angle θ between the incoming solar rays and the panels normal may be found by Equation 3, (refer to Figure 1):

$$\cos \theta = \cos(\beta) \cos(\psi - \phi) \text{ sine}(\Sigma) + \text{sine}(\beta) \cos(\Sigma) \qquad (3)$$

Where:

ψ = horizontal projection of the panel normal

ϕ = the solar azimuth

Σ = angle of tilt of the panel with the horizontal

The diffuse radiation falling on the surface is:

$$I_d = C\, I_{dn}\, F_{ss}\ [\text{Btuh/ft}^2] \qquad (4)$$

Where:

F_{ss} = angle factor between the surface and the sky

C = diffuse radiation factor (from Thvelkeld and Jordan)

The amount of solar radiation received during a 24-hr period for a clear day is obtained by integrating Equation 1 from sunrise to sunset:

$$E_d = \int_{t_u}^{t_d} I_t \, dt \qquad (5)$$

Where:

E_d = daily energy received (Btu/ft²/day)

t_u = hours from sun-up to solar noon

t_d = hours from solar noon to sundown

dt = time differential

The total amount of energy gained in one year, assuming all clear days is the sum of the daily totals:

$$E_y = \sum_{i=1}^{365} E_{d_i}\ [\text{Btu/ft}^2/\text{yr}] \qquad (6)$$

Where:

E_y = yearly energy, Btu/ft²/yr

i = day subscript

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*A. F. Souka and H. H. Satuat, "Optimum Orientations for the Double Exposure Flat-Plate Collector and Its Reflectors." Solar Energy, Vol. 10, No. 4, 1966. "Designing for Sun Power." Mechanical Engineering, 96:43, 54 5 '74.

†J. L. Thvelkeld and R. C. Jordan, "Direct Solar Radiation Available on Clear Days." ASHRAE Transactions, Vol. 64, 1958.

Panel shading

For panels in which L (the horizontal length for roof collector panels) is much greater than D (the distance between adjacent panels, the relationship governing the shading of a back or lower panel will be different for roof-mounted, southern facing and for side-mounted panels. The shading expressions can be derived from their trigonometric relationship. The derivation of the roof panel shading function is as follows (refer to Figure 2).

If the position of the sun is given to be at an altitude of β and an azimuth of ϕ, then the length of panel exposed to the direct sunlight is FS. From the law of sines:

$$\frac{\text{sine}\ (\beta)}{L_1} = \frac{\text{sine}\ (180 - \beta - \theta)}{L_2} \qquad (7)$$

Eliminating L_1, L_2 and θ yields:

$$FS = \frac{D \times \text{sine}\ \beta \cos\left[\tan^{-1}(\tan \Sigma \cos \phi)\right]}{\cos(\Sigma)\ \text{sine}\left[180 - \beta - \tan^{-1}(\tan \Sigma \cos \phi)\right]} \qquad (8)$$

Where D = distance between two rows of panels

Letting K be the ratio of L_3 (the length of unshaded panel inclined at angle sigma) to D; and letting L_3 equal one, the nondimensional value of FS is:

$$FS = \frac{\text{sine}\ \beta \cos\left[\tan^{-1}(\tan \Sigma \cos \phi)\right]}{K \cos \Sigma\ \text{sine}\left[180 - \beta - \tan^{-1}(\tan \Sigma \cos \phi)\right]} \qquad (9)$$

For uniform panels, FS is the fraction of direct radiation reaching the back panel. If shading is considered, the quantity of direct radiation reaching a back panel is:

$$I_{dir} = I_{dir} \times \cos \phi\ (\phi) \times FS \qquad (10)$$

Maximizing energy received

The foregoing equations were programed along with a Hooke and Jeeves direct search program*, which optimizes an objective function depending upon independent variables. In our case, the objective function U for a given panel or sets of panels is:

$U = U(x_1, x_2, \ldots x_n)$

$U(\Sigma, \psi, K)$

Where:

x_n = independent variables

n = number of independent variables

The independent variables in the search are then Σ, ψ, K and the objective function or the equation to be maximized is Equation 6.

This search also has the capability of evaluating the solution with regard to equality or inequality constraints. These are in the form:

$\psi j = (x_1, x_2, \ldots x_n) = 0 \quad j = 1, m$

$\phi_j = (x_1, x_2, \ldots x_n) < 0 \quad j = 1, p$

Where:

ψj = equality constraints

m = number of equality constraints

ϕ_j = inequality constraints

p = number of inequality constraints

The use of the above is shown in the following examples.

Example 1—Find angles Σ and ψ that give the maximum energy received for one year. The panel will be located on the roof with no shading present. For a given latitude the objective function for the search is:

$U = E_y(\Sigma, \psi)$

There are no constraints. The maximum value of U for a latitude of 40° is found from the direct search to correspond to the values:

$\Sigma = 34.9°$

$\psi = 0.0$

This was repeated for a variety of latitudes, and ψ was observed to be 0 in all cases. This was expected because of the symmetry of the sun's apparent motion with respect to the north-south plane. The value of Σ is plotted for various latitudes in Figure 3.

Example 2—Find the collector angle and optimum spacing for roof-mounted panels and the values of Σ and D

*J. N. Siddall, Analytical Decision-Making in Engineering Design, Prentice-Hall Inc. 1972.

FIGURE 1 ORIENTATION ANGLES OF SUN AND PANEL

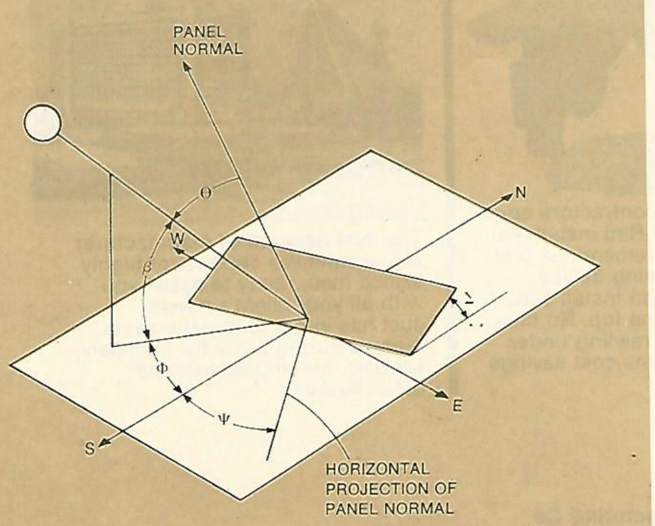


FIGURE 2
DETERMINATION OF ROOF PANEL SHADING FACTOR

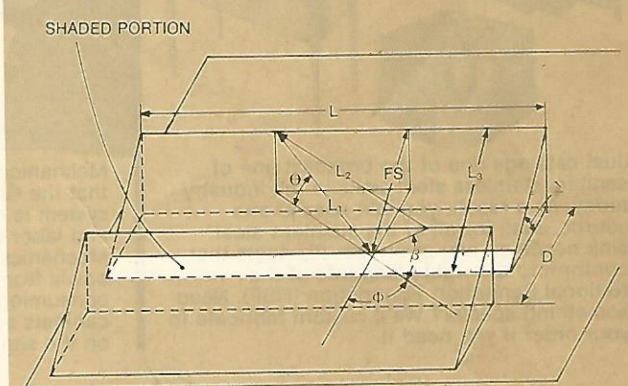
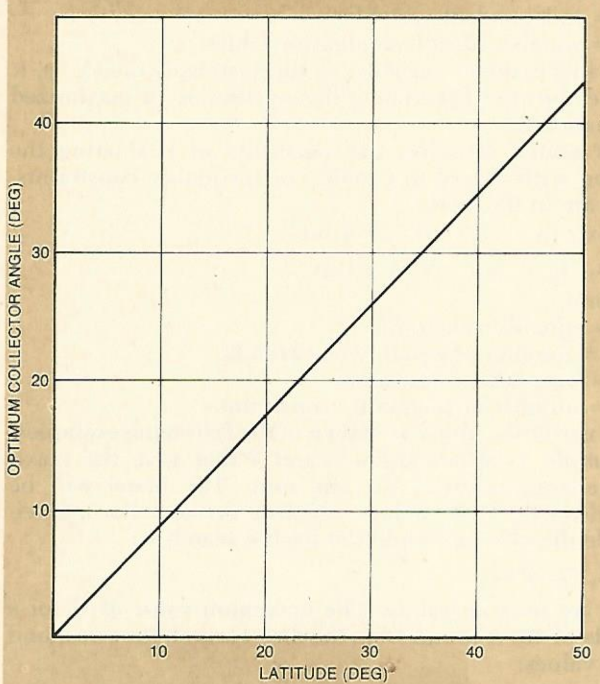


FIGURE 3 OPTIMUM COLLECTOR ANGLE



that will give the maximum yearly energy received per sq ft of roof that the panel is using. This value for the back panel in Figure 2 is D. If the panels overlap so that D is less than the $\cos\Sigma$, then the linear foot of roof per panel will be $\cos\Sigma$. The objective function is then:

$$U = E_y(\Sigma, D)/D \text{ if } D > \cos\Sigma$$
$$= E_y(\Sigma, D)/\cos\Sigma \text{ if } D < \cos\Sigma$$

The optimum values for a latitude of 40° are:

$\Sigma = 28.7°$

$D = .87$

Again, D is based on a unit panel length.

These examples represent only two situations where optimization is useful. There are numerous other cases that can be formulated and optimized, as in the case of panels mounted on the side- or front-face. For panels mounted on east or west faces of a building, it is structurally simpler to mount the panels in a vertical position. The independent variables for the search would then be ψ and D. Equation 9 would then be formulated to apply to side-mounted panels.

Variations of the objective function may also be used. Suppose panels are to be used mainly for summer cooling. The objective function would then sum the daily insolation values for the particular months only.

Constraints of many kinds may be used to fit the program to a specific problem. For esthetic reasons, the vertical height of the panel may be limited. Sine Σ would, therefore, be constrained to the desired value in the case of roof-mounted panels.

This program is useful for optimizing flat-plate collector angles and spacing and is flexible enough to apply to small or large panel installations and to a wide variety of applications. The user only need formulate the objective function and constraints to satisfy a particular problem. □